Name_____

CALCULUS AB/BC SUMMER REVIEW PACKET

Welcome to AP Calculus! Calculus is a branch of advanced mathematics that deals with problems that cannot be solved with ordinary algebra such as rate problems where slopes are not fixed and areas and volumes of irregular objects. Students will learn, through direct instruction and extensive practice, about the two main branches of calculus (differential and integral calculus) and related subjects.

One goal of this class is to provide students with the foundations for college science, math, and engineering classes. An essential component of this is the development of critical thinking skills, in particular, problem solving skills. As a result, we will emphasize the thought process – students will be expected not just to know the answers, but to explain them.

This is an "advanced placement" math class. Students in this class are expected to take the Calculus AP test. Students who perform well on this standardized test may be eligible to bypass introductory calculus classes at college. The exact amount of credit will depend on the test score, and on which college they choose to attend.

This is also considered to be a college-level course. As such, you will be treated more like college students. This means that more of the responsibility for your learning lies in your own hands than you have been accustomed to. By now, you should have a good sense of how to learn a complex subject. Do the work necessary and take the responsibility to seek help when you need it.

The problems in this packet are designed to help you review topics form algebra and pre-calculus which are important to your success in AP Calculus. All too often, students show perfect calculus work only to get the final answer incorrect due to algebra or trigonometry errors. Do yourself a favor and work out these pre-requisite skills at your leisure during the summer. When you come across a topic that requires a little review, feel free to search a website, call a friend, or e-mail me (pomeranced@fultonschools.org) for help.

This packet is your first homework assignment of the upcoming school year. Understand, however, that time will not be spent in class this fall going over these questions. The expectation is that you will enter the classroom in August with this packet complete and ready to start new material.

Check your answers on the provided answer key.

We are looking forward to working with you in the fall. Have a great summer!

David Pomerance – pomeranced@fultonschools.org

I. Simplify. Identify the zeros, vertical asymptotes, horizontal asymptotes, holes and sketch each rational function. Show the work that leads to your graph.

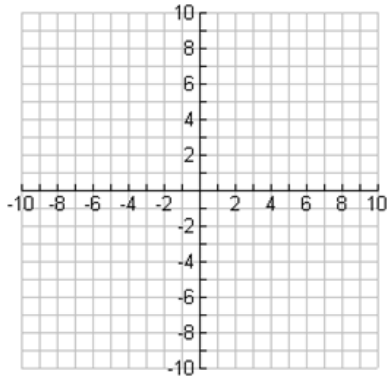
1) $f(x) = \frac{x^3 - x}{2x^3 - 8x}$

Zeros: _____

VA: _____

HA: _____

Holes: _____



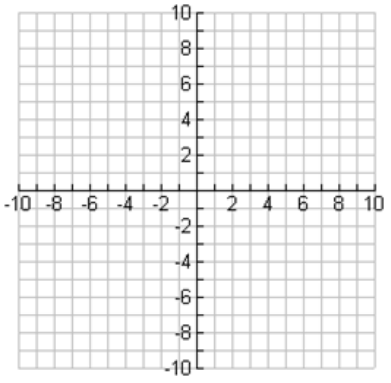
2) $f(x) = \frac{x^2 - 4x - 32}{x^2 - 16}$

Zeros: _____

VA: _____

HA: _____

Holes: _____



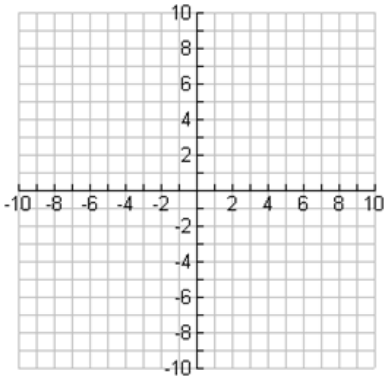
3) $f(x) = \frac{x - 4}{x^2 - 3x - 4}$

Zeros: _____

VA: _____

HA: _____

Holes: _____



Show work:

II. Complete the following identities:

1) $\sin^2 x + \cos^2 x =$ _____

2) $\cos^2 x =$ _____

3) $\cot^2 x + 1 =$ _____

4) $\sin^2 x =$ _____

5) $\sin(2x) =$ _____

6) $\cos(2x) =$ _____

7) $\tan^2 x + 1 =$ _____

_____

_____

III. Simplify each expression:

1) $\frac{1}{x+h} - \frac{1}{x}$

2) $\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$

3) $\frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

4) $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

5) $\frac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

6) $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

7) $\frac{x}{\sqrt{x+9}-3}$
(Hint: Rationalize the denominator)

8) $\frac{\sqrt{x+h}-\sqrt{x}}{h}$
(Hint: Rationalize the numerator)

IV. Solve for z:

1) $4x + 10yz = 0$

2) $y^2 + 3yz - 8z - 4x = 0$

V. Expand and Simplify:

1) $\sum_{n=0}^{4} \frac{n^2}{2} =$

2) $\sum_{n=1}^{3} \frac{1}{n^3} =$

VI. Determine each of the following given that…

$f(x) = \{(3,5), (2,4), (1,7)\}$
$h(x) = \{(3,2), (4,3), (1,6)\}$

$g(x) = \sqrt{x-3}$
$k(x) = x^2 + 5$

1) $(f \circ g)(3) =$

2) $(g \circ k)(7) =$

3) $\frac{1}{f(x)} =$

4) $(kg)(x) =$

5) $(f + h)(1) =$

6) $(k - g)(5) =$

7) $f^{-1}(x) =$

8) $k^{-1}(x) =$

VII. Miscellaneous: Follow directions for each problem.

1) Evaluate $\frac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

2) Expand $(2x - y)^5$ using Pascal's Triangle.

3) Simplify $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$

4) Eliminate the parameter and write a rectangular equation for $x = t^2$ and $y = 2t$.

VIII. Simplify:

1) $\frac{\sqrt{x}}{x}$

2) $e^{\ln 3}$

3) $e^{1+\ln x}$

4) $\ln 1$

5) $\ln e^{7}$

6) $\log_3\left(\frac{1}{3}\right)$

7) $\log_{\frac{1}{2}} 8$

8) $\left(16a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

9) $e^{3\ln x}$

10) $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

11) $27^{\frac{2}{3}}$

12) $\left(5a^{\frac{2}{3}}\right)\left(4a^{\frac{3}{2}}\right)$

Write the following expressions as a single fraction.

13) $a^{-3} - \frac{1}{2}a^{-2} + \frac{3}{4}a^{-1}$

14) $(x+1)^{-1/2} + (x+1)^{1/2} \cdot x$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line:

1) with slope -2, containing the point $(3,4)$ _____

2) containing the points $(1,-3)$ and $(-5,2)$ _____

3) with slope 0 , containing the point $(4,2)$ _____

4) perpendicular to the line $3y + 2x = 6$, containing the point $(3,4)$ _____

5) parallel to the line $y - 3x = 7$, containing the point $(3,4)$ _____

X. Without a calculator, determine the exact value of each expression:

1) $\sin \pi$
2) $\sin \frac{\pi}{2}$
3) $\sin \frac{3\pi}{4}$
4) $\cos 0$
5) $\cos \frac{3\pi}{4}$
6) $\cos \frac{\pi}{3}$
7) $\tan \frac{7\pi}{4}$
8) $\tan \frac{\pi}{6}$
9) $\cot \frac{5\pi}{6}$
10) $\sec \frac{4\pi}{3}$
11) $\csc \frac{2\pi}{3}$
12) $\cos\left(sin^{-1} \frac{1}{2}\right)$
13) $\arcsin 1$
14) $\arctan 1$
15) $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$

XI. Determine all points of intersection:

1) Line $x + y = 8$ and line $4x - y = 7$

2) Parabola $y = x^2 + 3x - 4$ and line $y = 5x + 11$

3) Parabola $y = 4 - x^2$ and parabola $y = x^2 + 2x$

XII. For #1-6, determine the function's domain and range. For #7-10, evaluate.

$f(x) = x - 4$ $g(x) = x^2 - 4$ $h(x) = \ln x$

Function	Domain	Range
1) $\ln\big(f(x)\big)$		
2) $\sqrt{g(x)}$		
3) $f\big(g(x)\big)$		
4) $g\big(f(x)\big)$		
5) $h(x)$		
6) $h^{-1}(x)$		

7) $g\big(f(7)\big)$

8) $f^{-1}\big(g(2)\big)$

9) $h(1)$

10) $h^{-1}(1)$

XIII. Solve for x, where x is a real number. Show the work that leads to your solution.

1) $x^2 + 3x - 4 = 14$

2) $\frac{x^4-1}{x^3} = 0$

XIV. Given the vectors v = -2i + 5j and w = 3i + 4j , determine:

1) $\frac{1}{2}\mathbf{v}$

2) $\mathbf{w} - \mathbf{v}$

3) length of **w**

XV. Express each of these fractions as the sum of two or more fractions with simpler denominators. (This is called partial fraction decomposition.)

1) $\frac{x}{x^2+5x+6}$

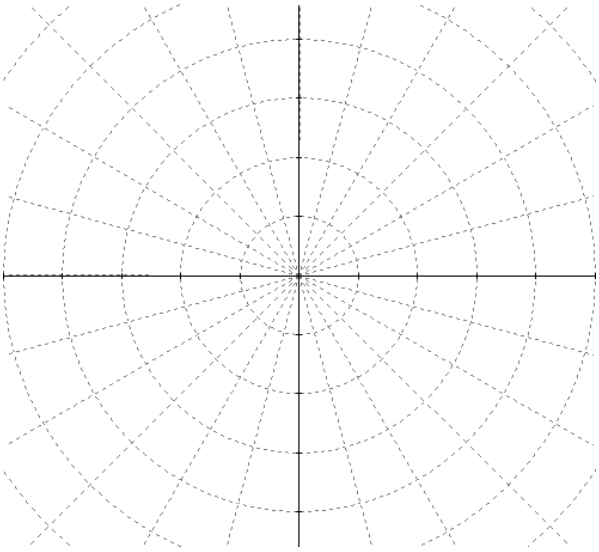
2) $\frac{1}{x^2-3x+2}$

3) $\frac{2x^2+2}{4-x^2}$

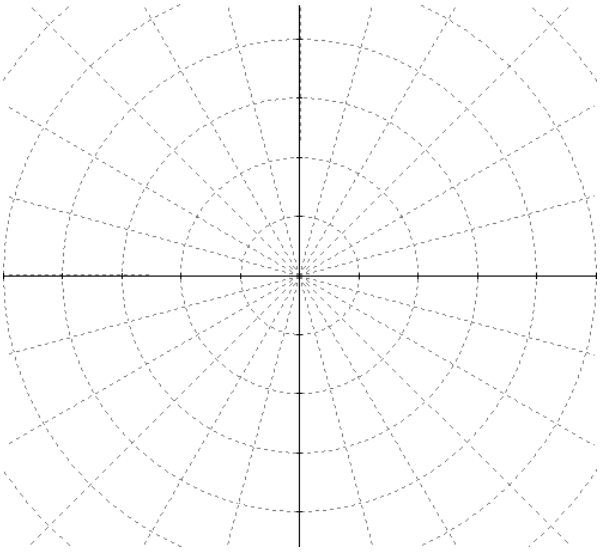
4) $\frac{x^3+x^2+x}{x^2+2x+1}$

XVI. Sketch a graph of each.

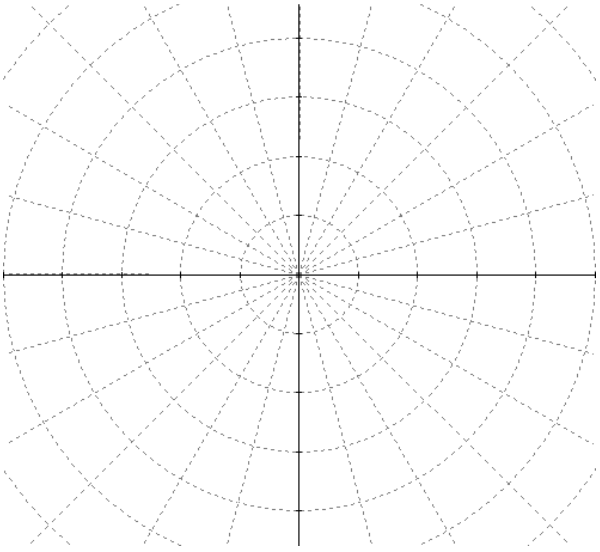
1) $r = 2$



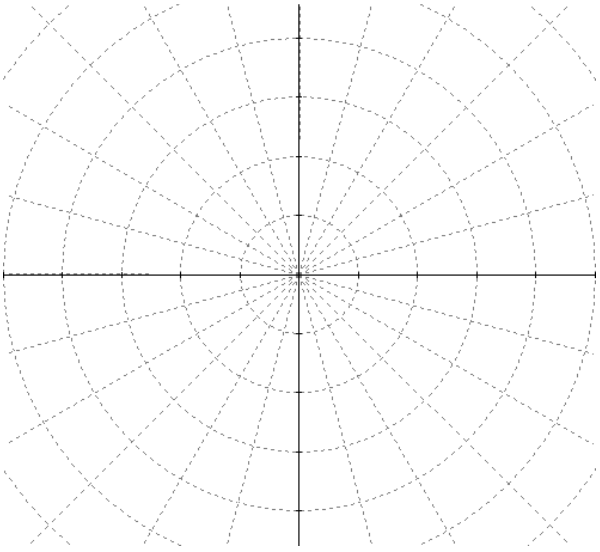
2) $r = 3\sec\theta$



3) $r = 1 + \sin\theta$

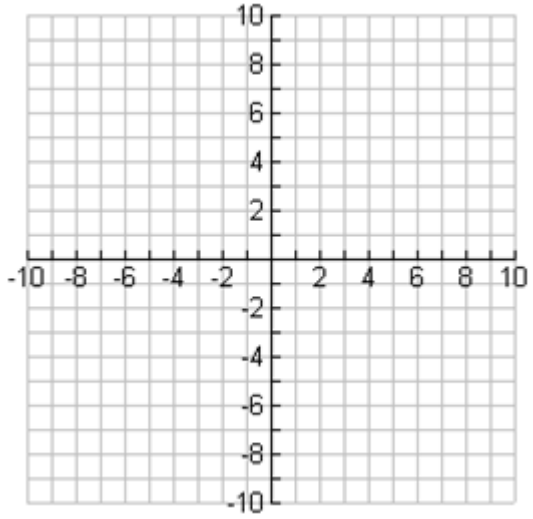


4) $r = 2\cos(3\theta)$



XVII. Graph each function. Give its domain and range.

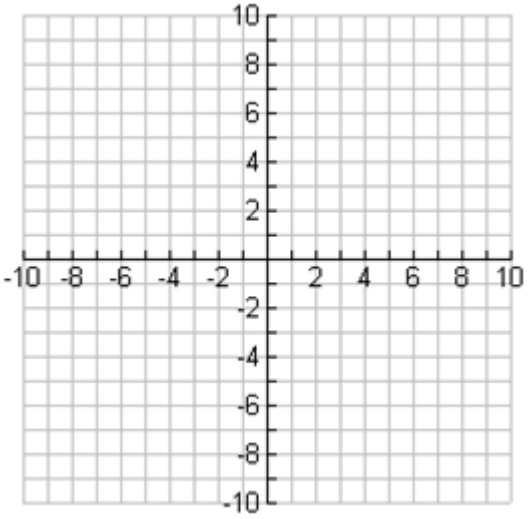
1) $y = \sin x$



Domain: _____

Range: _____

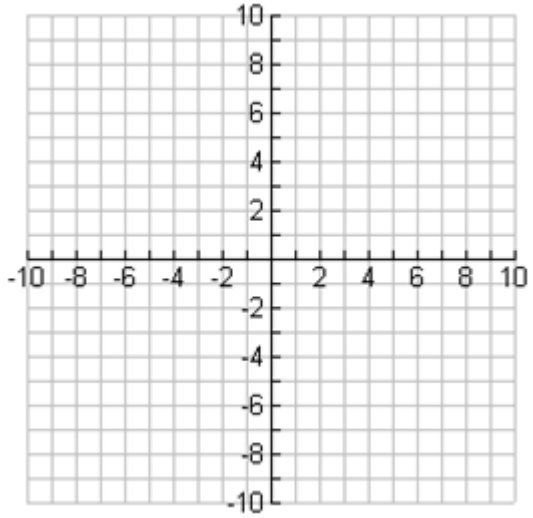
2) $y = e^x$



Domain: _____

Range: _____

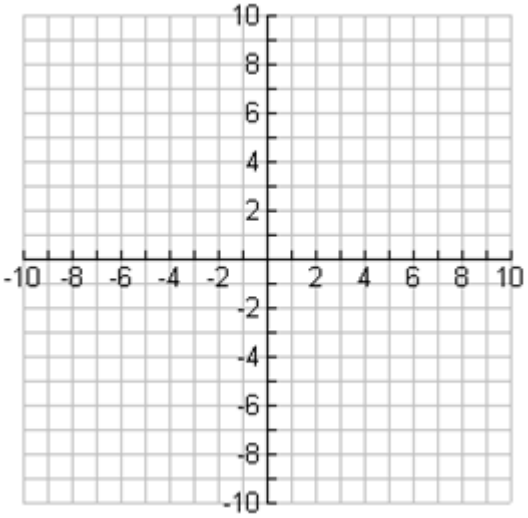
3) $y = \sqrt{x}$



Domain: _____

Range: _____

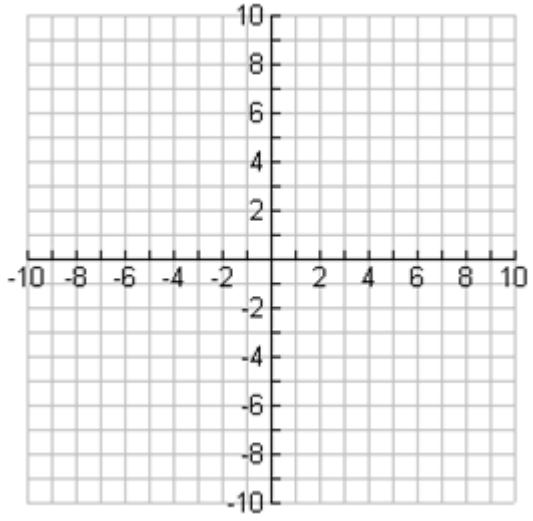
4) $y = \sqrt[3]{x}$



Domain: _____

Range: _____

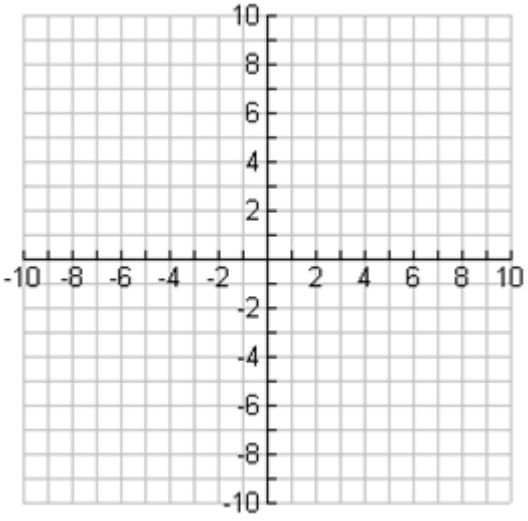
5) $y = \ln x$



Domain: _____

Range: _____

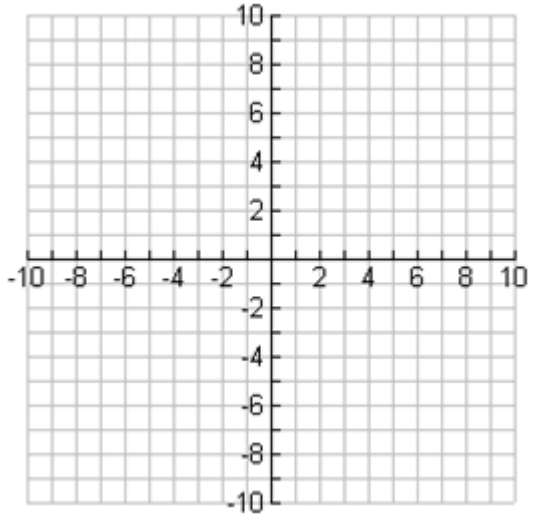
6) $y = |x + 3| - 2$



Domain: _____

Range: _____

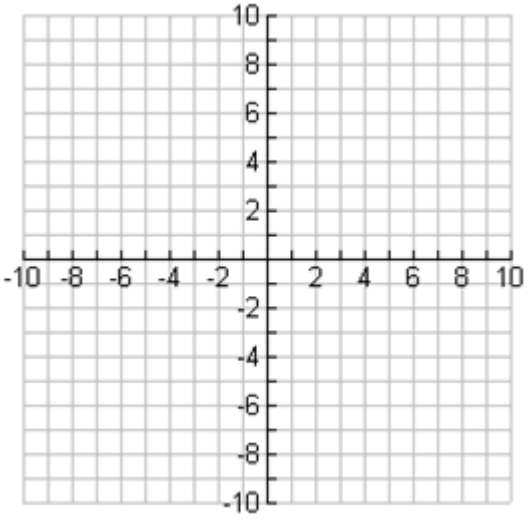
7) $y = \frac{1}{x}$



Domain: _____

Range: _____

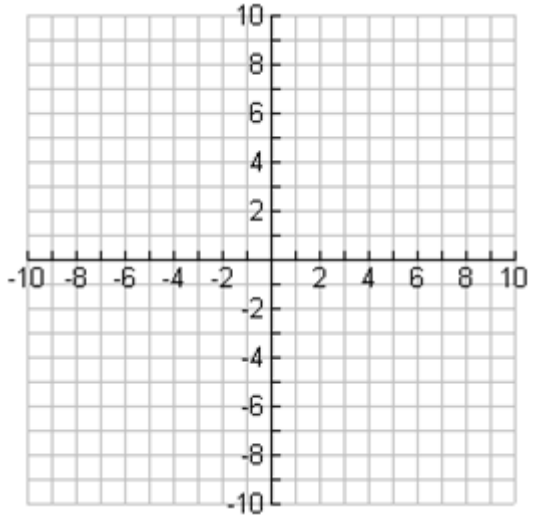
8) $y = \begin{cases} x^2 & x < 0 \\ x + 2 & 0 \leq x \leq 3 \\ 4 & x > 3 \end{cases}$



Domain: _____

Range: _____

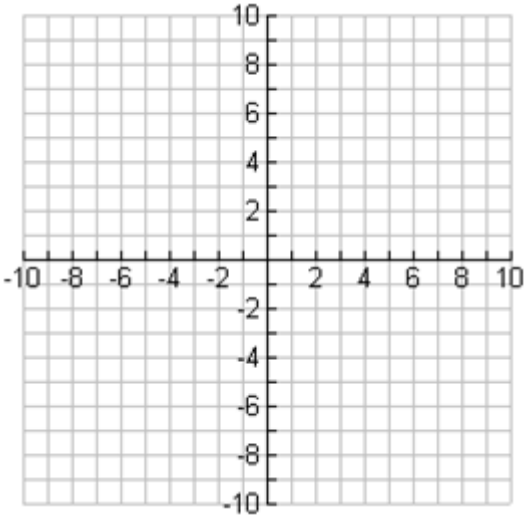
9) $y = \frac{1}{x^2}$



Domain: _____

Range: _____

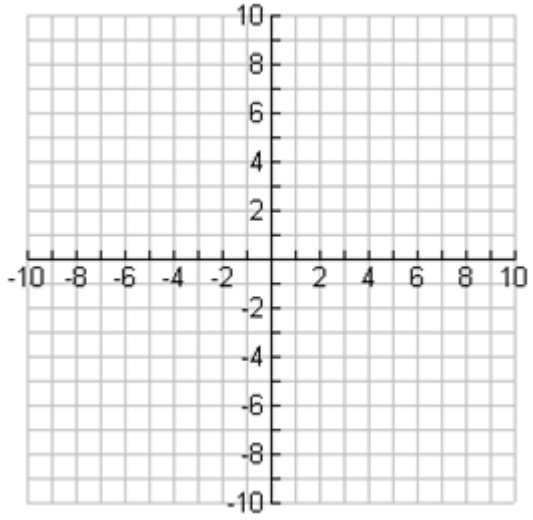
10) $y = \csc x$



Domain: _____

Range: _____

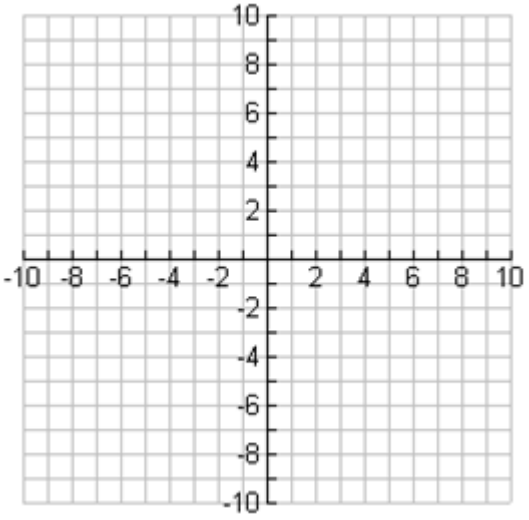
11) $y = (x + 2)^3 - 4$



Domain: _____

Range: _____

12) $y = \tan x$

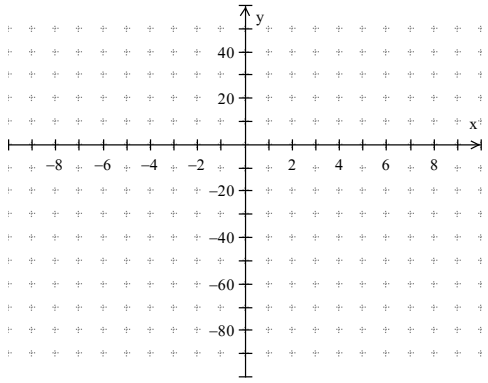


Domain: _____

Range: _____

XVIII. Answer the following questions with a graphing calculator. All answers should be accurate to 3 decimal places.

1) In your calculator, graph $f(x) = 2x^4 - 11x^3 - x^2 + 30x$ using a window size as follows: x min = -10, x max = 10, y min = -100, y max = 60. Sketch the result below.



2) Find all roots (zeros) of $f(x)$.

3) Find all local maxima of $f(x)$.

4) Find all local minima of $f(x)$.

5) Find the intervals over which $f(x)$ is positive.

$(-\infty, -1.5) \cup (0,2) \cup (5, \infty)$

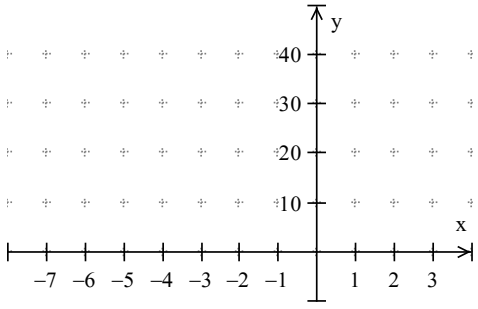
6) Find the intervals over which $f(x)$ is increasing.

$(-0.890, 1.067) \cup (3.948, \infty)$

7) Use the table function in your calculator to complete the table.

x	-2	-1	-0.2	1	1.25	2.218	4.947
$f(x)$							

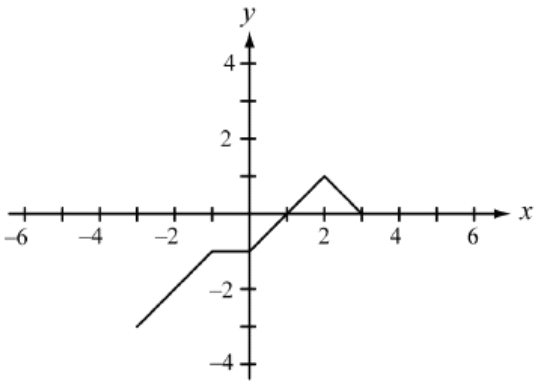
8) In your calculator, graph $g(x) = x^3 + 5x^2 - 7x + 2$ and $h(x) = 0.2x^2 + 10$ using a window size: x min = -8, x max = 4, y min = -10, y max = 50. Sketch the result below.



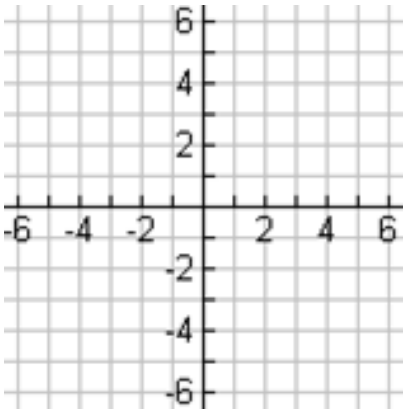
9) Find all points of intersection of $g(x)$ and $h(x)$.

XIX. Function *f*, defined on the closed interval $[-3,3]$, is graphed to the lower left:

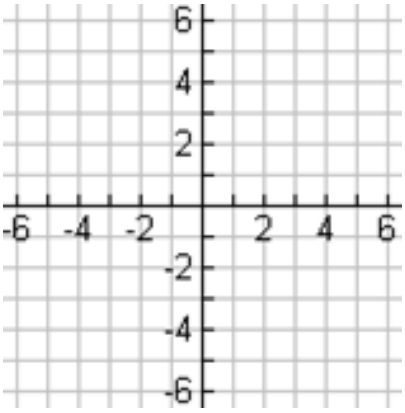
Graph of $y = f(x)$



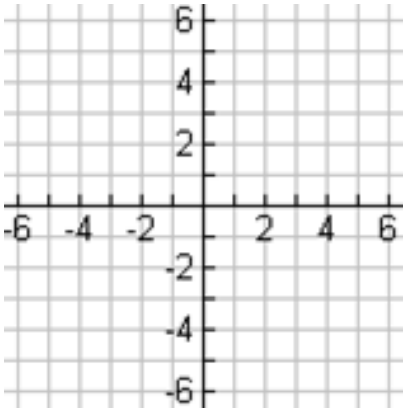
1) Sketch $y = |f(x)|$.



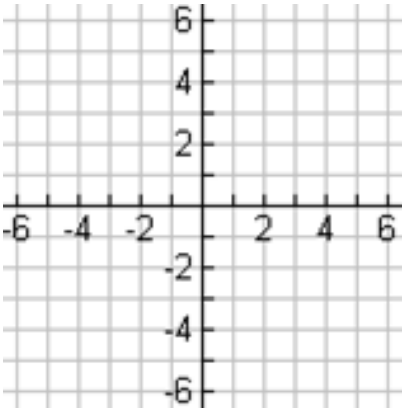
2) Sketch $y = f(-x)$.



3) Sketch $y = f\left(\frac{x}{2}\right)$.



4) Sketch $y = f(x - 1) + 2$



5) Sketch $y = f(|x|)$.